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Final Report

THE VARIABLE POLARITY PLASMA ARC WELDING PROCESS:
MATHEMATICAL MODEL OF WELDING SYSTEM

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### ABSTRACT

Significant advantages of Variable Polarity Plasma Arc (VPPA) welding process include faster welding, fewer repairs, less joint preparation, reduced weldment distortion, and absence of porosity. In this report, a mathematical model is presented to analyze the VPPA welding process. Results of the mathematical model have been compared with the experimental observation accomplished by the GDI team.

#### I. INTRODUCTION

The plasma arc is a concentrated energy source commonly used in welding and cutting processes. It is composed of a partially ionized gas stream produced by forcing an inert gas to flow through an electric arc and emerge from a constricting nozzle. With its high energy density and velocity, the plasma arc, when impinging on the workpiece, can create a hole in the molten pool generated by its heat and can penetrate through the workpiece. Depending on the operating parameters employed, this hole may either become selfhealing or remain open as the arc transverses along the workpiece. A "keyhole" welding process occurs in the selfhealing case, in which the molten metal in front of the arc flows around the arc column and resolidifies behind the arc. On the other hand, a cutting operation is achieved if the hole remains open (O'Brien, 1968).

The plasma welding process is essentially an extension of gas tungsten-arc welding (GTAW); however, constraining the arc to flow through a nozzle produces much higher energy density in the arc and much higher gas velocity and momentum. The process produces deep penetration welds by forming a keyhole in the workpiece by the pressure of the gas flow. The metal melted in front of the advancing keyhole flows around to the rear where it solidifies to form the weld bead. This is distinct from the essentially surface melting produced by the GTAW process (which cannot normally penetrate to a depth equal to the width of a weld pool).

In 1955 Linde Air Products introduced a plasma arc torch for metal cutting applications, and by 1965 Linde had developed an automatic plasma arc welding facility for Westinghouse Electric Corporation to fabricate a 120 in. diameter, 3/8 in. thick D6AC steel rocket motor case for the Titan III-C booster assembly (Miller and Filipski, 1966; Privoznik and Miller, 1966). The plasma arc welding process was reported to halve the welding time required. 1965 Thermal Dynamics Corporation reported the use of direct current reversed polarity (workpiece negative) plasma arc to join 1/4 in. thick aluminum plate (Cooper et al., 1965). the end of the 1960's, Van Cleave at the Boeing Company began his efforts to combine the plasma arc process with a variable polarity feature in which the electrode polarity was periodically reversed. Alternating electrode potential for aluminum welding had been investigated as early as 1947 (Herbst, 1947). Difficulties with welding power supplies in this application were evident early-on. In addition, when variable polarity plasma arc (VPPA) welding was used in the U.S. Army Roland Missile Production Program, development problems such as arc pressure pulsation were noted (Nunes et al., 1981; 1983). As a result of Van Cleave's promising work at Boeing, a VPPA research and development project was initiated in 1978 at the NASA Marshall Space Flight Center, to determine the potential for replacing the GTAW system used in the fabrication of the Space Shuttle External Tank (ET). After the original used in the fabrication and process were

improved, VPPA welding finally exceeded the expectations (Nunes et al., 1984a; 1984b).

The Space Shuttle ET (diameter 28.6 ft., length 154 ft.) is the largest known "drop tank," carrying 140,000 gallons of liquid oxygen and 380,000 gallons of liquid hydrogen. From the outset of ET production, the conventional GTAW system operating in the direct current electrode negative mode was used. With the GTAW system, in spite of the requisite careful joint preparation, weld porosity and defects had been ever-present problems since the fabrication of the Saturn lunar rocket first stage. Therefore, the decision was made at the outset to use 100% radiographic inspection. Weld porosity and defects had to be ground out and repaired systematically.

The VPPA process significantly reduced porosity and defects with less stringent joint preparation and, in addition, reduced weldment distortion, and speeded up the welding process.

In this study, the power distribution for an argon plasma gas has been analyzed along its course through the VPPA welding system. The study includes the following sections:

- (1) Electric potential distribution of straight and reverse polarities, and potential intensities for the VPPA welding system;
- (2) VPPA welding system electric power input at the electrode, within orifice, in the free plasma jet column and in the keyhole of workpiece;

- (3) Power loss at electrode, within the gap between electrode and within orifice, standoff column and the workpiece;
- (4) Determination of plasma arc enthalpy;
- (5) Determination of crown and root bead widths;
- (6) Determination of crown and root heights;
- (7) Determination of leading edge angle of keyhole.
- (8) Comparison between molding and experimental measurements

# II. VARIABLE POLARITY PLASMA ARC WELDING SYSTEM ELECTRIC POTENTIAL CALCULATION

The significant current carriers in a welding arc are electrons and positive ions. Electrons carry the bulk of the current, moving rapidly from negative cathode to positive anode. The positive ions drift more slowly through the interelectrode space. In a fixed polarity arrangement, the differential drift rate results in an asymmetrical heating of the welding arc's ends. The cathode receives less heat and the anode more heat. The straight polarity mode of operation entails a negative electrode (cathode) and a positive workpiece (anode). Where the primary object of the weld process is to deliver the maximum heat to the workpiece with minimum deterioration of the electrode, straight polarity is used.

Reverse polarity, which entails positive electrode and negative workpiece, has the advantage that the workpiece is subjected to a cleaning process (cathodic cleaning) by the impingement of positive ions on the workpiece surface. Recent work suggests that heavy positive ion weight is irrelevant to the cleaning process which seems to be more like capacitor breakdown than hammer impact. In the case of plasma arc welding, reverse polarity action appears to condition the surface of the aluminum alloy so that the molten metal flows easily and controllably under the arc. It is conjectured that this fluid control is accomplished through breaking up of surface oxide films. Cutoff of reverse

polarity during VPPA welding transforms a weld metal flow which closes smoothly and soundly behind the keyhold to an irregular, intermittent, globular flow. This process leaves a rough, lumpy bead sunken below the parent metal surface and protruding jaggedly from the root of the weld. However, continuous reverse polarity is not necessary to provide adequate cathodic cleaning action.

The variable polarity square-wave with unequal straight and reverse polarity time intervals offered a combination of the high heating capability of straight polarity with the cleaning feature of reverse polarity. Adequate cleaning is obtained by incorporating a relatively short (1/10 to 1/5 the duration of the straight polarity current) pulse of reverse polarity current into the welding current waveform.

Figure 1 show the VPPA electric power system. During the straight polarity mode of operation, it employs a negative electrode (cathode) and a positive workpiece (anode); during the reverse polarity mode of operation it employs a positive electrode and a negative workpiece. Figure 1 at left illustrates the straight polarity mode; at right, reverse polarity mode.

Calculation of electric potential can be obtained from the following formulations:

#### (A) For Straight Polarity

$$V_E = V_F - \Delta V_a - \phi_W$$

$$V_{G} = V_{E} - \sum_{i} \left(\frac{dV}{dL}\right)_{3,i} \Delta L_{3,i}$$

$$V_c = V_G - \sum_{i} \left(\frac{dV}{dL}\right)_{2,i} \Delta L_{2,i}$$

$$V_D = V_C + \Delta V_a + \phi_N$$

$$V_B = V_C - \sum_{i} \left(\frac{dV}{dL}\right)_{1,i} \Delta L_{1,i}$$

$$V_{A} = V_{B} - \Delta V_{C} + \phi_{E}$$

$$\Delta V_{m}^{+} = \Delta V_{FA} = V_{F} - V_{A} = -V_{A}$$

$$\Delta V_{p}^{+} = \Delta V_{DA} = V_{D} - V_{A}$$

where  $\phi_E$  = Electrode Work Function

 $\phi_{N}$  = Orifice Work Function

 $\phi_{\mathbf{W}}$  = Workpiece Work Function

 $\Delta V_a = Anode Drop$ 

 $\Delta V_c$  = Cathode Drop

 $\Delta L_{1,i}$ ,  $\Delta L_{2,i}$ ,  $\Delta L_{3,i}$  = Length of segment i in  $L_{1}$ ,  $L_{2}$ , and  $L_{3}$ , respectively

## (B) For Reverse Polarity

$$V_E' = V_F' + \Delta V_C - \phi_V + \epsilon_R$$

$$V_{G'} = V_{E'} + \sum_{i} \left(\frac{dV}{dL}\right)_{3,i} \Delta L_{3,i}$$

$$V_{C'} = V_{G'} + \sum_{i} \left(\frac{dV}{dL}\right)_{2,i} \Delta L_{2,i}$$

$$V_{D'} = V_{C'} - \sum_{i} \left(\frac{dV}{dL}\right)_{2,i} \Delta L_{2,i} + \Delta V_{a} + \phi_{N}$$

$$V_{B'} = V_{C'} + \sum_{i} \left(\frac{dV}{dL}\right)_{1,i} \Delta L_{1,i}$$

$$V_{A'} = V_{B'} + \Delta V_{a} + \phi_{E}$$

 $\epsilon_R$  = Reverse Polarity Rise = A + B  $\frac{t_-}{t_+ + t_-}$ 

t<sub>+</sub> = Straight Polarity Time Period

t\_ = Reverse Polarity Time Period

$$\Delta V_{m}^{-} = \Delta V_{\lambda}, F_{F} = V_{\lambda}, - V_{F}, = V_{\lambda}.$$

$$\Delta V_{p}^{-} = \Delta V_{A} \cdot p \cdot = V_{p} \cdot - V_{A} \cdot$$

#### (C) For Potential Intensities

Electric potential intensities of the plasma fluid in the plasma column can be calculated from the electric conductivity,  $\sigma_E$ , of plasma fluid at the location of interest. The value of electric conductivity at the location of interest is determined by the local plasma jet pressure and temperature. By using the data provided by Devoto (1973), Lancaster (1986) has calculated the electrical conductivity of argon plasma gas as it varies with temperature at one atmosphere pressure. Figure 2 shows the variation of argon gas electrical conductivity with temperature at one atmospheric pressure. In this model

calculation, we have divided the lengths of plasma column along the axial direction into several segments with  $\Delta L_i$  as the length of segment i. Joule heating, and radiative and convective heat transfer losses at the segment i are calculated with the assumed value of electric potential intensity of that segment,  $(\Delta V/\Delta L)_i$ , and the given inlet values of power and enthalpy of plasma fluid at that segment. Outlet values of power and enthalpy of plasma fluid at that segment are then calculated based on these parameters. The value of electric conductivity  $\sigma_{E,i}$  at segment i is thus determined from the average fluid temperature at that segment. The value of average temperature is obtained from the corresponding average value of plasma fluid enthalpy at that segment which is calculated from the average value of inlet and outlet plasma fluid enthalpys at that segment. Electrical resistance,  $R_i$ ; electric field difference,  $\Delta V_i$ ; and electric field intensity,  $(\Delta V/\Delta L)_i$ , at segment i can be determined from the computed value of electric conductivity  $\sigma_{E,i}$  at that segment. Convergent solution is obtained through the trial and error compuations between the computed value of  $(\Delta V/\Delta L)$ ; and the assumed value of  $(\Delta V/\Delta L)_i$  at segment i. From these calculations, it is clearly shown that the values of electric potential of the plasma fluid and heat transfer computations are mutually coupled together. Furthermore, the values of plasma electrical conductivity fluctuate with local variations of

temperature. The physical parameters affecting the plasma temperature, such as plasma and shielding gas flow rates, radiative and convective heat transfer losses, values of main and pilot electric currents, standoff distance, etc., all influence the values of electric potential and electric field intensity in the plasma fluid.

The effect of shielding gas flow rate occurs through shielding gas mixing into the plasma standoff column  $L_3$  and vice versa. The continuous mixing between plasma gas and shielding gas induced by the plasma jet stream in plasma column results in local variations of species constituent concentration and temperature along the axial direction. Thus, the effect of jet mixing in column streams between plasma and shielding gases must be taken into consideration in addition to heat transfer losses in computing electric potential along the plasma jet. Figure 3 shows the variation of electrical resistance with temperature for various argon-helium mixtures.

III. VPPA WELDING SYSTEM ELECTRIC POWER INPUT

VPPA Electric Power Input can be calculated as follows:

$$P_{\text{total}} = \frac{I_{m}^{+} \Delta V_{m}^{+} t_{+} + I_{p}^{+} \Delta V_{p}^{+} t_{+} + I_{m}^{-} \Delta V_{m}^{-} t_{-} + I_{p}^{-} \Delta V_{p}^{-} t_{-}}{t_{+} + t_{-}}$$
(3-1)

where Im + = Straight Polarity Main Current

Ip + = Straight Polarity Pilot Current

I<sub>m</sub> = Reverse Polarity Main Current

I<sub>p</sub> = Reverse Polarity Pilot Current

Equation (3-1) is the total electric power input of the system which is the summation of power input for the plasma arc jet in the gap between electrode and orifice, power input within orifice, power input in free jet column (standoff), and power input within keyhole of workpiece. The power input can be itemized, and calculated as follows:

(III-A) Power Input at the Electrode

$$P_{E} = \frac{(I_{m}^{+} + I_{p}^{+})(\Delta V_{c} - \phi_{E})t_{+} + (I_{m}^{-} - I_{p}^{-})(\Delta V_{k} + \phi_{E})t_{-}}{t_{+} + t_{-}}$$
(3-2)

(III-B) Power Input in the Gap Between Electrode and Orifice
(Joule Heating)

$$P_{J,1} = \frac{(I_m^+ + I_p^+) \left[ \sum_{i} \left( \frac{dV}{dL} \right)_{1,i} \Delta L_{1,i} \right] t_+ + (I_m^- - I_p^-) \left[ \sum_{i} \left( \frac{dV}{dL} \right)_{1,i} \Delta L_{1,i} \right] t_-}{t_+ + t_-}$$
(3-3)

(III-C) Power Input Within Orifice

This includes power input through the work function of metal

$$P_{N,1} = \frac{I_p^+ \phi_N t_+ + I_p^- \phi_N t_-}{t_+ + t_-}$$
 (3-4)

Power input through anode drop,

$$P_{N,2} = \frac{I_p^+ \Delta V_a t_+ + I_p^- \Delta V_a t_-}{t_+ + t_-}$$
 and power input through Joule heating within the orifice

Joule heating within the orifice

$$P_{J,2} = \frac{I_{m}^{+} \left[ \sum_{i} \left( \frac{dV}{dL} \right)_{2,i} \Delta L_{2,i} \right] t_{+} + (I_{m}^{-} - I_{p}^{-}) \left[ \sum_{i} \left( \frac{dV}{dL} \right)_{2,i} \Delta L_{2,i} \right] t_{-}}{t_{+} + t_{-}}$$
(3-6)

# (III-D) Power Input in the Free Plasma Jet Column (Standoff) (Joule Heating)

$$P_{J,3} = \frac{I_m^{+} \left[ \sum_{i} \left( \frac{dV}{dL} \right)_{3,i} \Delta L_{3,i} \right] t_{+} + I_m^{-} \left[ \sum_{i} \left( \frac{dV}{dL} \right)_{3,i} \Delta L_{3,i} \right] t_{-}}{t_{+} + t_{-}}$$

$$(3-7)$$

It is important to note that shielding gas surrounding the plasma jet, is induced into the plasma jet column due to the flow generated by the plasma jet stream. Power of shielding gas introduced into the plasma jet, Psd, shown in Equation (3-29), shall be included as a part of the plasma jet power input, in addition to the power input shown in Equation (3-7).

### (III-E) Power Input in the Keyhole of Workpiece

This includes power input through the work function of metal and reverse polarity potential rise

$$P_{w,1} = \frac{I_m^+ \phi_v t_+ + I_m^- (\epsilon_R - \phi_v) t_-}{t_+ + t_-}$$
 (3-8a)

and power input through anode and cathode drops

$$P_{w,2} = \frac{I_m^+ \Delta V_a t_+ + I_m^- \Delta V_c t_-}{t_+ + t_-}$$
 (3-8b)

# (III-F) Total Power Input to the VPPA Welding System

Total power input of VPPA welding system, shown in Equation (3-1) shall be equal to the summation of power input shown in Equation (3-2) to (3-8) and plasma jet induced shielding gas power input,  $P_{sd}$ , namely  $P_{total} = P_E + P_{J,1} + P_{N,1} + P_{N,2} + P_{J,2} + P_{J,3} + P_{W,1} + P_{W,2} + P_{sd}$  (3-9)

# IV. POWER LOSS CALCULATION OF VPPA WELDING SYSTEM (IV-A) Electrode Heat Conduction Loss

A one-dimensional formulation of the electrode heat conduction loss is given by

$$Q_E = k_E \cdot A_E \frac{(T_E - T_0)}{L_E}$$
 (4-1)

where

Q<sub>E</sub> = Electrode Heat Conduction Loss

 $k_E$  = Thermal Conductivity Coefficient of Electrode

 $A_{r}$  = Cross-sectional Area of Electrode

L<sub>E</sub> = Length of Electrode (from tip to water-cooled collet)

 $T_E - T_o = Temperature Difference of Electrode Between Tip and Collet Position$ 

## (IV-B) Radiation Heat Loss Between Electrode and Orifice

Plasma arc heat radiation loss within the gap between electrode and orifice can be computed from the following equation (Evans and Tankin, 1967; Cram, 1985):

$$Q_{G}(W) = V_{G}(m^{3}) (4 \times 10^{10}) \left[ \frac{T(^{0}K)}{15000} \right]^{16} / \left\{ 1 + \left[ \frac{T(^{0}K)}{15000} \right]^{16} \right\}$$
 (4-2)

where

 $Q_{G}$  = Heat Radiation Loss Within the Gap Between Electrode and Orifice (W)

 $V_G$  = Plasma Arc Jet Volume Within Gap Between Electrode and Orifice (m<sup>3</sup>)

T = Absolute Temperature (°K)

(IV-C) Power Loss Within the Orifice

Power Loss within the orifice is comprised of the following items:

## (IV-C-1) Heat Convection Loss Within the Orifice

Heat convection loss within the orifice can be computed from the following formulation of convective heat transfer model (Hsu and Rubinsky, 1987):

$$Nu = 0.2233(X^+)^{-0.7455} + 3.66$$
 (4-3)

where

$$X^{+} = \frac{X}{R_{0}} \cdot \frac{1}{(Re \cdot Pr)_{m}}$$

Here

X = Axial Distance, (half distance from the entrance of orifice
to the outlet end of the segment of interest)

 $R_0$  = Radius of Orifice

Re = Reynolds Number of Plasma Arc Fluids

Pr = Prandtl Number of Plasma Arc Fluids

Nu = Nusselt Number of Plasma Arc Fluids

Subscript m = mean values

Heat convection loss within the orifice, thus, can be computed from the Nusselt number obtained from Equation (4-3), namely,

$$Q_{N,c} = Nu \cdot A_N \cdot \frac{k}{D} \cdot \frac{(h_{plasma} - h_N)}{c_p}$$
 (4-4)

where

 $Q_{N,c}$  = Heat Convection Loss within the Orifice

 $A_N$  = Heat Transfer Area of Segment of Interests

D = Diameter of Orifice =  $2 R_0$ 

k = Thermal Conductivity of Plasma Arc

c<sub>p</sub> = Constant Pressure Specific Heat of Plasma Arc

 $h_{plasma}$  = Average Enthalpy of Plasma Arc through the Orifice

h<sub>N</sub> = Average Enthalpy of plasma in Orifice corresponding
to the Orifice Wall Temperature

All thermal properties used in this model are determined at the temperature corresponding to the average enthalpy of plasma arc.

## (IV-C-2) Direct Power Loss to Surface of Orifice

Direct power loss to the orifice surface includes  $P_{N,1}$ , orifice work function induced power input; and part of  $P_{N,2}$ , orifice anode drop induced power input, namely

$$Q_{N,G} = P_{N,1} + \psi P_{N,2} \tag{4-5}$$

where,

 $Q_{N,G}$  = Direct power loss from the orifice surface

 $\psi$  = Fraction power generated at by anode drop lost to orifice  $(1-\psi)P_{N,2}$  = Plasma arc power input from power generated at orifice anode drop

# (IV-C-3) Electron Collision Power Loss Within Orifice

Power loss due to free electrons entering the surface of orifice can be calculated from the following equation:

$$Q_{N,e} = \frac{I}{e} \left( \frac{3}{2} KT \right) \tag{4-6}$$

where

 $Q_{N,e}$  = Free electron power loss to the orifice

e = Electron charge =  $1.602 \times 10^{-19}$  coulomb/mole

 $K = Boltzman constant = 1.38 \times 10^{-16} e^{rg}/_{mol\cdot K}$ 

T = Plasma arc temperature;

and I is the electric current which can be computed as follows:

$$I = \frac{I_p^+ t_+ + I_p^- t_-}{t_+ + t_-}$$
 (4-7)

Total power loss in orifice,  $Q_N$ , is the summation of Equations (4-4), (4-5), and (4-6), namely,

$$Q_{N} = Q_{N,c} + Q_{N,G} + Q_{N,e} (4-8)$$

## (IV-D) Power Loss from Standoff Column

Power loss from the standoff column includes radiative heat loss,  $Q_{s,r}$ , and convective heat loss,  $Q_{s,c}$ .

Calculation of radiative heat loss is similar to Equation (4-2) for that in the gap between electrode and orifice, namely

$$Q_{s,r}(W) = V_{s}(m^{3}) (4 \times 10^{10}) \left[ \frac{T({}^{0}K)}{15000} \right]^{16} / \left\{ 1 + \left[ \frac{T({}^{0}K)}{15000} \right]^{16} \right\}$$
 (4-9)

where  $V_{\S}$  denotes plasma arc volume within the standoff column; and T is the absolute temperature corresponding to the average value of plasma arc enthalpy.

Radiative heat loss is strongly dependent on fluid temperature. This and what follows require trial and error computations.

To compute the convective heat loss,  $Q_{\$,c}$ , from the standoff column, jet stream mixing theory is employed. Turbulent mixing between plasma jet and the surrounding shielding gas is shown schematically in Figure 4.  $r_1$  stands for the inner boundary of jet boundary layer, and  $r_2$  expresses the outer boundary of jet boundary

layer. Both are functions of axial distance x from the outlet end of the orifice. The inner jet core region, surrounded by  $r_1(x)$  is a non-perturbed zone of jet core. The jet boundary layer lies between  $r_2(x)$  and  $r_1(x)$ . Jet stream parameters in the jet boundary layer, including velocity, temperature and species concentration, vary from original (pure plasma gas jet) parameters at  $r = r_1$  to ambient (pure shielding gas) parameters at  $r = r_2$ . The jet core extends to  $x = x_H$ . As  $x > x_H$ , the jet core disappears and the jet boundary layer occupies the entire extent from r = 0 to  $r = r_2$ . The initial region from x = 0 to  $x = x_H$  is the region of jet mixing. Boundary thickness at transition  $x = x_H$  is  $r_2$ ,  $x_1$ .

Based on the formulations of Abramovich (1963), the boundary layer within the initial region of jet mixing is governed by the following equations:

$$\frac{R_0 - r_1}{cx} = \frac{1 - m}{1 + m} \left[ 0.416 + 0.134m + 0.021 \frac{cx}{R_0} \left( \frac{1 - m}{1 + m} \right) \cdot (1 + 0.8m - 0.45m^2) \right]$$
(4-10)

$$\frac{R_0 - r_1}{r_2 - r_1} = 0.416 + 0.134m + 0.021 \frac{(r_2 - r_1)}{R_0} (1 + 0.8m - 0.45m^2)$$
(4-11)

where  $R_0$  denotes the radius of orifice;  $r_1$ , the radius of inner boundary of jet boundary layer;  $r_2$ , the radius of outer boundary of jet boundary layer; and x, the axial distance from the outlet end of orifice to the point of interest.  $m = u_H/u_0$ , where  $u_H$  is ambient fluid velocity; ;and  $u_0$ , the original jet flow velocity inside jet core. c is empirical constant which is 0.27 when  $u_0 >> u_H$ .

Substituting the values of m=0 and c=0.27 in Equations (4-10) and (4-11), we have

$$\frac{R_0 - r_1}{x} = 0.27 \left( 0.416 + 0.00567 \frac{x}{R_0} \right)$$
 (4-12)

$$\frac{R_0 - r_1}{r_2 - r_1} = 0.416 + 0.021 \left(\frac{r_2 - r_1}{R_0}\right)$$
 (4-13)

From equation (4-12) , we can solve for the jet inner boundary  $r_1$  in term of the axial distance x which is given by

$$\frac{r_1}{R_0} = 1 - 0.1123 \frac{x}{R_0} - 0.0015 \left(\frac{x}{R_0}\right)^2$$
 (4-14)

Similarly, from Equation (4-13), we can determine the boundary layer thickness,  $r_2 - r_1$ , i.e.,

$$r_2 - r_1 = \frac{1}{2} \{-19.81R_0 + [392.44R_0^2 + 190.48 (R_0 - r_1) R_0]^{1/2}\}$$
(4-15)

To determine the axial distance of jet initial region  $x_H$ , we can solve Equation (4-14) by substituting  $r_1 = 0$ , i.e.,

$$\frac{x_H}{R_0} = 8 \tag{4-16}$$

In our case, the radius of orifice  $R_0 = 1.5875$  mm, and the axial distance of the jet initial region  $x_H = 8(1.5875) = 12.7$  mm. In the present study, the range of standoff distance considered is between 1 to 10 mm, so that all standoff values considered in the present study lie within the range of the initial region of jet mixing. Let us consider velocity, temperature and species concentration profiles within the range of jet initial region as follows: